Predicting vibration levels on an experimental test case by using invariant loads (e.g. blocked forces) as source characterization

J. Ortega Almirón1,2, F. Bianciardi1, P. Corbeels1, W. Desmet2,3
1 Siemens Industry Software NV, Interleuvenlaan 68, B-3001 Leuven, Belgium
e-mail: jesus.ortega@siemens.com
2 KU Leuven, Department Mechanical Engineering, Celestijnenlaan 300 B, B-3001, Heverlee, Belgium
3 DMMS core lab, Flanders Make, Celestijnenlaan 300c, 3001 Leuven, Belgium

Abstract
Component-based TPA is a relatively new TPA approach that allows to characterize a structureborne source component independently from the receiver structure (as opposed to classical TPA) and to predict its behavior when coupled to different receivers. This approach would allow to frontload the development process and considerably increase the flexibility during the design process. However, there are a number of challenges affecting its applicability, such as a proper modelling of the coupling degrees of freedom at the interface, the difficulty to access the interface connection points and the conditioning of the equations. A number of innovative methods were developed to address these issues, that will be investigated by means of measurements on a laboratory test bench. A systematic analysis will be performed through which the accuracy and applicability of the component-based TPA will be analyzed and assessed.

1 Introduction

Classical Transfer Path Analysis (TPA) is a well-established technique that successfully identifies the transmission paths of noise and vibration problems [1][2][3]. One of the requirements of classical TPA is the physical availability of the vehicle to estimate the structural and acoustic loads from operational measurements.

Due to the increasing vehicle variants, shorter development cycles, the trend to reuse the same components on all vehicle variants and the limited availability of prototypes, the automotive industry is seeking for alternative technologies to predict the contribution of vehicle components (engine, intake, exhaust, tyres, etc.) upfront, early in the development process. In that sense, it is necessary the development and validation of techniques that allow the prediction of the contributions from the different sources, without actually being assembled in the final vehicle. However, even though in theory it would be possible to calculate those contributions from the internal load and structural data of the vehicle, in practice is a challenging procedure whether because of the high complexity of the internal load generation mechanisms inside the source or the difficulty to access it to obtain the data in a proper and accurate way. With this purpose, the sources (components that comprise the basic internal load generator mechanisms) are characterized by a set of pseudoforces or blocked forces [4], [5]. These quantities represent in a unique way the load exerted by the source and transmitted to any receiver structure. As opposed to classical TPA, component-based TPA tries to characterize those quantities. This procedure can be performed with the source installed on a test bench or directly on a vehicle, namely in-situ source characterization [6].
The characterization of the source is combined with the structural data of both, source and receiver, for the contribution prediction. Frequency Based Substructuring (FBS) is a method used to calculate the acoustic and structural behavior of the assembly using FRF data from the source and receiver structures separately [7]. FBS was first introduced by Bishop and Johnson [8] and became widely known with the approach of Jetmundsen et al. [9].

FBS requires to access the defined DOFs of the connections, this can be achieved by applying geometrical reduction. This technique allows to obtain the required DOFs without actually measure in the exact locations [10], what would be useful when the locations are difficult or impossible to access, and when moments and rotations are needed (especially important for rigid connections), but at the expense of using more impacts and responses for the modelling.

This paper aims to validate every step of this full process, from the source characterization on a test-bench to the structural vibration prediction in a simplified vehicle receiver. Strengths and weaknesses of its practical implementation are highlighted in an academic setup.

2 Methods for source characterization and vibration prediction

2.1 Source characterization: pseudoforces and blocked forces

To characterize a source is necessary to obtain a quantity that, by itself, allows the full description of the source for a given operational condition, as well as the prediction of its behavior when coupled to any known receiver structure. Classical TPA can identify the contact forces that are transmitted between the source and the receiver, however, these forces are dependent on the assembled structure (source + receiver) and cannot be transferred to different receivers. Therefore contact forces don’t successfully identify the source independently [11].

Component-based TPA is a relatively new TPA technique aiming to independently characterize a source by a set of forces (pseudoforces or blocked forces), based on the hypothesis of a finite number of connecting DOFs between source and receiver [12].

It’s known that a certain source with an input force $F_i$ will generate a field of accelerations $\mathbf{a}_r$ in the receiver that it is connected to, that depends on both, source and receiver, according to eq. (1).

$$\mathbf{a}_r = Y_{SR}^{fS} F_i$$

Where $Y_{SR}^{fS}$ is the frequency response function of the coupled system between the input forces of the source and any point $r$ in the receiver. (Variables are expressed in the frequency domain). The system is depicted in figure 1.

![Figure 1: Source and receiver system model](image)

Input forces are used here to refer the external or internal forces that the basic source mechanism exerts on the source component. If we assume that those input forces $F_i$ are always independent from the receiver, these forces would already be a valid characterization of the source. However, this way is discarded in practice, as most of the sources generate an input force that is infeasible or impossible to measure accurately.
because of the complexity of the multiple mechanisms that might be involved and the impossibility to access them.

To tackle that problem one may use the general concept of pseudoforces. For a source assembled on a receiver with a finite number of connecting DOFs, \( n \), a set of \( n \) forces can be calculated for a certain set of \( n \) points in the source, such that it would generate the same accelerations in the \( n \) DOFs of the connection when the source is turned off, as the source would do when active (fig 2(a)) [13]. Those forces can be easily calculated with matrix inversion of a square matrix, as long as it is well conditioned (eq. (2)).

\[
F_{ps} = Y_{csp}^{-1} a_c = Y_{csp}^{-1} Y_{SR} F_i
\]

(2)

Where \( F_{ps} \) is the set of pseudoforces, \( a_c \) is the set of accelerations in the connection DOFs, \( Y_{csp} \) is the frequency response function of the coupled system between the pseudoforces and the connection, and \( Y_{SR} \) is the frequency response function of the coupled system between the input forces and the connection. When the calculated set of forces act in the opposite way, the generated accelerations in the connections would also be the opposite to the ones generated by the source. Therefore when both, source and the opposite set of calculated forces, act at the same time, the accelerations at the connection DOFs are cancelled, resulting on the connection DOFs remaining still (fig 2(b)). As the receiver is only connected through those DOFs, not having any additional source, all the points of the receiver will also remain still, hence the accelerations of the receiver can be generated by the same set of forces, namely pseudoforces. This only holds for receiver accelerations and cannot be extended to accelerations in the source.

That means that pseudoforces can be calculated using the accelerations of any DOF that belongs to the receiver, either in the connection or not, for any connected receiver. Therefore, they can also be calculated using the accelerations of the connections of the uncoupled source in operational conditions, as it can be assumed to be coupled to a massless, infinitely flexible receiver, according to eq. (3).

\[
F_{ps} = Y_{rps}^{-1} a_r = Y_{rps}^{-1} Y_{SR} Y_{csp}^{-1} Y_{Ci} F_i
\]

(3)

Where \( Y_{rps} \) is the frequency response function of the coupled system between the pseudoforces and any point \( r \) in the receiver, \( Y_{csp} \) is the frequency response function of the decoupled source between the pseudoforces and the connection, and \( Y_{Ci} \) is the frequency response function of the decoupled source between the input forces and the connection. It can be noticed that all the possible sets of pseudoforces would generate the same acceleration field in the receiver but not in the source. Also different sets of pseudoforces would generate different acceleration fields in the source.

Figure 2: (a) Pseudoforces acting alone; (b) opposite pseudoforces acting with the source in operation

When the pseudoforces are calculated in the connection DOFs, they are called blocked forces [14]. Therefore blocked forces are a particular case of pseudoforces with the same properties (fig 3(a) and 3(b)).
Figure 3: (a) Blocked forces acting alone; (b) opposite blocked forces acting with the source in operation

As they are calculated in the connection (eq. (4)), they are the same forces that the source would exert to a blocking restriction in the connection DOFs. However, this theoretical definition is not a feasible way to measure them in practice, as the ideal blocking restriction is physically impossible to reproduce for most sources, especially without affecting the source dynamics. Thus, in order to calculate blocked forces, \( F_{bl} \), of a source mounted on a receiver structure, either in-situ characterization or bench dynamics compensation has to be applied [15].

\[
F_{bl} = Y_{rc}^{SR}a_r^{-1}
\]  

(4)

Where \( Y_{rc}^{SR} \) is the frequency response function of the coupled system between the connection and any point \( r \) in the receiver.

### 2.2 Frequency based substructuring

The pseudoforces or blocked forces need to be combined with coupled FRF data, experimental or simulated, in order to predict the final contribution in the vehicle. Coupled FRFs can be experimentally measured, if the components are physically available. When only the uncoupled components are available, FBS is applied to calculate the FRFs of the coupled setup from the FRFs of the uncoupled source and receiver [16].

Let us take the subsystem’s FRF matrices and model them as an uncoupled system in eq. (5). In this system, the effect of the coupling is represented with a set of external forces \( g \) acting in the connections of both bodies as contact forces, as shown in fig 4. The contact forces must guarantee the compatibility condition of the connection DOFs, represented in eq. (6), and the equilibrium condition of eq. (7).

\[
\begin{align*}
\mathbf{a} &= \begin{bmatrix}
\mathbf{a}^S_s & \mathbf{a}^S_c & \mathbf{a}^R_s & \mathbf{a}^R_c
\end{bmatrix} = \begin{bmatrix}
Y^S_{ss} & Y^S_{sc} & 0 & 0 \\
Y^S_{cs} & Y^S_{cc} & 0 & 0 \\
0 & 0 & Y^R_{rr} & Y^R_{rc} \\
0 & 0 & Y^R_{cr} & Y^R_{rr}
\end{bmatrix} \begin{bmatrix}
F^S_s \\
F^S_c \\
F^R_s \\
F^R_c
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
g^c \\
g^r
\end{bmatrix} = Y(f + g) \\
\mathbf{a}^R_c - \mathbf{a}^S_c = [0 \ -I \ 1 \ 0]a = Ba = 0
\end{align*}
\]

(5)

(6)

In these and following equations of this section the subscripts make reference to the DOFs (s: any source DOFs, sc: connection in source side, rc: connection in receiver side, r: any receiver DOFs, c: connection, si: input forces, sps: pseudoforces) and the superscripts make reference to the component (S: source, R: receiver, SR: coupled source and receiver). The equilibrium equation uses \( \lambda \) as the Lagrange multipliers in order to guarantee the condition. Substituting eq. (7) in (5) and in (6) an expression for \( \lambda \) can be obtained in eq. (8).

\[
\lambda = -(BYB^T)^{-1}BYf
\]

(8)

Using this expression in eq. (7) and (5) leads to the general FBS equation, eq. (9).

\[
Y^{SR} = Y - Y(BYB^T)^{-1}BY
\]

(9)
When the connection between source and receiver is not rigid, but relative displacements are allowed by means of a mount of certain stiffness, a similar derivation can be carried out, with a different equation (eq. (10)) in the place of the previous compatibility and equilibrium conditions.

\[ g = -B^T K (a^R_{rc} - a^F_{sc}) = -B^T K B a \]  

(10)

Where \( K \) is the stiffness of the mount. The new coupled acceleration matrix for this case is shown in eq. (11).

\[ Y^{SR} = Y - YB^T (K^{-1} + BYB^T)^{-1} BY \]  

(11)

We can expand those expressions to look at the equations in terms of DOFs as in eq. (5), obtaining eq. (12).

\[ Y^{SR} = \begin{bmatrix} Y^S_{SS} \\ Y^S_{SC} \\ Y^S_{CR} \\ Y^R_{CS} \\ Y^R_{CC} \\ Y^R_{CR} \end{bmatrix} \]

\[ \begin{bmatrix} 0 \\ 0 \\ 0 \\ Y^R_{RC} \end{bmatrix} \]

\[ \begin{bmatrix} \begin{bmatrix} Y^S_{SS} - Y^S_{SC} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{csi} \\ Y^S_{CS} - Y^S_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{csi} \\ Y^S_{CR} - Y^S_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{csi} \\ Y^R_{CS} - Y^R_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{cpsi} \\ Y^R_{CC} - Y^R_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{cpsi} \\ Y^R_{CR} - Y^R_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{cpsi} \end{bmatrix} \end{bmatrix} \]

(12)

Let us take the first column of the coupled FRFs and particularize it to the DOFs of the input forces of the source mechanism to calculate accelerations in the coupled system (eq. (13)). When the input force is substituted by the definition of pseudoforce (eq. (3)) eq. (14) is obtained.

\[ \begin{bmatrix} a^S_s \\ a^S_sc \\ a^R_{rc} \\ a^S_r \end{bmatrix} = \begin{bmatrix} Y^S_{SS} - Y^S_{SC} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{csi} \\ Y^S_{CS} - Y^S_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{csi} \\ Y^S_{CR} - Y^S_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{csi} \\ Y^R_{CS} - Y^R_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{cpsi} \end{bmatrix} F_i \]

(13)

\[ \begin{bmatrix} a^S_s \\ a^S_sc \\ a^R_{rc} \\ a^S_r \end{bmatrix} = \begin{bmatrix} Y^S_{SS} - Y^S_{SC} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{cpsi} \\ Y^S_{CS} - Y^S_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{cpsi} \\ Y^S_{CR} - Y^S_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{cpsi} \\ Y^R_{CS} - Y^R_{cc} [Y^S_{cc} + Y^R_{cc} + K^{-1}]^{-1} Y^S_{cpsi} \end{bmatrix} F_i \]

(14)

It can be noticed that the resultant set of FBS equations also supports the pseudoforce assumptions described in the previous section. The last 3 lines of eq. (14) are the exact same FBS equations of the coupled FRF system with the pseudoforces as input. Moreover, the last 2 lines prove that the accelerations in the receiver of the coupled system can be calculated using pseudoforces combined with coupled FRFs. As expected, the first line of eq. (14) indicates that is not possible to calculate the accelerations in the source using pseudoforces combined with coupled FRFs, neither is to obtain pseudoforces using source accelerations as indicators. Unique exception is to use source accelerations at the connections, as proven in the second line of eq. (14).

2.3 Geometrical reduction

The coupling of the components using FBS and the blocked forces calculation usually require the access to input and output DOFs that cannot be directly measured, whether because of feasibility issues or just nonexistence of the material point. Also a finite number of DOFs is required by FBS and blocked forces practicability. For that purpose the surrounding region of the point of interest is assumed to exhibit a rigid behavior, creating a rigid body dependence between the DOFs contained by the region. A schematic geometrical reduction (GR) is shown in fig 5.
That dependence allows to transform the measured forces and accelerations into the forces and accelerations of any geometrical point that is assumed to fulfill the rigid behavior of the region, which is used to represent a rigid body. Note that it is not required that the representing point exists as a real point of the body or is positioned within its boundary. The number of measured DOFs is reduced to the final point by using geometrical relationships.

Let us consider the measured accelerations, \( \mathbf{a}_i \), in the region of interest assumed to be rigid. This region will be reduced to the point \( \text{GR}_i \), that represents the geometrical reduction \( i \). There will be a set of 6 geometrically reduced rigid DOFs \( q_i \) which allow to approximate the measured DOFs with eq. (15).

\[
\mathbf{a}_i = \mathbf{R}_a q_i = \begin{bmatrix} a_{i1} \\ \vdots \\ a_{iN_a} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{a1} \\ \vdots \\ \mathbf{R}_{aN_a} \end{bmatrix} q_i
\]  

(15)

Where \( a_{ij} \) is the acceleration of the DOF \( j \) for the geometrical reduction \( i \) and \( \mathbf{R}_{ai} \) is a matrix that contains the geometrical relationships of the rigid body behavior of the acceleration DOF \( j \). \( N_a \) is the number of acceleration DOFs used for the geometrical reduction \( i \). Each acceleration can be calculated using the expanded expression of eq. (16).

\[
\mathbf{a}_i^j = \mathbf{R}_{ai}^j q_i = \begin{bmatrix} e_x^j & e_y^j & e_z^j & r_x^j & r_y^j & r_z^j \\ q_x^i \\ q_y^i \\ q_z^i \\ q_{Rx}^i \\ q_{Ry}^i \\ q_{Rz}^i \end{bmatrix}
\]

(16)

Where \( r^j \) are the distances of the acceleration DOF \( j \) respect to \( \text{GR}_i \) for each direction, and \( e^j \) are the projections of the DOF \( j \) direction to each of the unitary vectors. It can be noticed that to be able to calculate \( q_i \) in an inverse manner, a minimum number of 6 equations are needed to construct the \( \mathbf{R}_{ai} \) matrix, however, a higher number is recommended in order to overdetermine the system [17]. Those singular equations are assembled for every individual geometrical reduction, which in turn can be assembled to form the global geometrical reduction system of the component, as shown in eq. (17).

\[
\mathbf{a} = \mathbf{R}_a q = \begin{bmatrix} a_1 \\ \vdots \\ a_{N_i} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{a1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{R}_{aN_i} \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_{N_i} \end{bmatrix}
\]

(17)

Where \( N_i \) is the number of individual geometrical reductions. This equation system allows to calculate the reduced acceleration DOFs \( q \) by matrix inversion using the pseudoinverse of \( \mathbf{R}_a \) with eq. (18).

\[
q = (\mathbf{R}_a^T \mathbf{R}_a)^{-1} \mathbf{R}_a^T \mathbf{a} = \mathbf{T}_a a
\]

(18)
A similar reasoning can be applied to the input DOFs. When a set of forces is applied on a region that is assumed to be rigid, the resultant force reduced to any point can be calculated using analogous geometrical relationships, as shown in eq. (19).

\[ m_i = R_{fi} f_i = \begin{bmatrix} R_{fi1} & \cdots & R_{fin} \end{bmatrix} \begin{bmatrix} f_{i1} \\ \vdots \\ f_{in} \end{bmatrix} \quad (19) \]

With \( R_{fi} \) being a matrix that contains the geometrical relationships of the rigid body behavior for the input DOF \( j \) and \( m_i \) being the resultant reduced forces for the geometrical reduction \( i \). \( N_{fi} \) is the number of force DOFs for the geometrical reduction \( i \). Every force \( f_{ij} \) generates a set of 6 resultant forces in the region according to eq. (20).

\[
\begin{bmatrix}
m_x^i \\
m_y^i \\
m_z^i \\
m_{Rx}^i \\
m_{Ry}^i \\
m_{Rz}^i
\end{bmatrix} = \sum_{j=1}^{N_{fi}} R_{fi}^j f_{ij} = \sum_{j=1}^{N_{fi}} \begin{bmatrix} e_x^j \\ e_y^j \\ e_z^j \\ r_{y}^j e_x - r_{z}^j e_y \\ r_{z}^j e_x - r_{x}^j e_z \\ r_{x}^j e_y - r_{y}^j e_x \\ \end{bmatrix} f_{ij}^j 
\quad (20)
\]

It can be noticed that to be able to calculate \( f_i \) in an inverse manner, a minimum number of 6 equations are needed to construct the \( R_{fi} \) matrix, however, a higher number is recommended in order to overdetermine the system. Those singular equations are assembled for every individual geometrical reduction, which in turn can be assembled to form the global geometrical reduction system of the component, as shown in eq. (21).

\[
m = R_{fi} f = \begin{bmatrix} m_1 \\ \vdots \\ m_{N_{fi}} \end{bmatrix} = \begin{bmatrix} R_{f1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_{fi_{N_{fi}}} \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_{N_{fi}} \end{bmatrix} 
\quad (21)
\]

This equation system allows to calculate one possible set of \( f \) that could generate the resulting reduced forces \( m \) by matrix inversion using the pseudoinverse of \( R_{fi} \), as shown in eq. (22).

\[
f = R_{fi}^{-T} (R_{fi} R_{fi}^{-T})^{-1} m = T_{fi} m 
\quad (22)
\]

Both acceleration and forces geometrical reductions can be applied to the FRF matrix of the system to calculate \( Y_{GR} \), the reduced FRF matrix, according to eq. (23).

\[
a = Y f = R_{a} q = YT_{fi} m \rightarrow q = T_{a} YT_{fi} m = Y_{GR} m 
\quad (23)
\]

This technique will be applied in the experimental setup to both forces and accelerations, in the connections, to properly accomplish the FBS (fig. 6) and to calculate blocked forces.

Figure 6: Geometrical reduced DOFs for an FBS
3 Experimental results

3.1 Test setup

For the experimental validations, three components are combined and tested in two setups. One component is used as a vehicle resemblance structure, made mainly by steel bars and a cavity. A second component is considered as a test bench to characterize the source. A star shaped aluminum plate constitutes the source. The sensors and bolts are considered part of the components in order to avoid mass loading effects. For all the components the FRFs are measured in coupled and uncoupled conditions using and impact hammer.

The first setup, consisting of the source on the test bench (fig 7(a)), is used to characterize the source by a set of blocked forces. The second setup, consisting of the source assembled with the vehicle structure (fig 7(b)), is used to validate the results of the component-based TPA process. As input force, a closed loop controlled mini shaker (resembling the source mechanism) exciting frequencies between 20 and 1500 Hz, is used.

The FRFs used to calculate the blocked forces are obtained by applying geometrical reduction from the impact points close to the connection to the center of each connection. However, in both setups and the uncoupled source, the impact points for FRFs measurement at the connection are in the same locations, so the geometrical reduction to calculate the blocked forces is the same for both setups. This means that the blocked forces transfer is equivalent to a pseudoforces transfer, as this geometrical reduction doesn’t have an influence on the prediction.

The source is connected with each receiver with 3 bolted connections, which are modelled as punctual connections. As the connections are rigid and restrict all the displacements, 6 DOFs per connection are considered in the coupling. In this way we try to characterize the complete coupling interface, which is needed for an adequate model [18]. To calculate reduced forces and accelerations, in each connection of both, source and receiver structures, 3 triaxial accelerometers and 6 impact points are used per connection (fig 8(a) and 8(b)). Additionally, 9 extra indicators are instrumented in the uncoupled test bench for a total of 36 indicators, to achieve an overdetermination of 2 in the blocked forces calculation. 3 targets instrumented in the vehicle structure (T1, T2, T3 in fig 7(b)) are used for validation purposes. The steps to validate the process are schematically depicted in figure 9.
Figure 8: Vehicle setup impacts (red dots and lines) and accelerometers location close to the connections. (a) Uncoupled. (b) Coupled.

Figure 9: Schematic description of the component-based TPA process steps. (left) Test bench. (right) Vehicle structure.

To properly validate the results, improve stability and optimize the method, different sets of DOFs (presented in table 1) are used to calculate the blocked forces, and the obtained results are compared.

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>All indicators</td>
<td>The 36 DOFs indicators in the receiver, not GR</td>
</tr>
<tr>
<td>Without Rz</td>
<td>Excluding the DOFs of moments in Z direction for each connection</td>
</tr>
<tr>
<td>Without Rx Ry Rz</td>
<td>Excluding the DOFs of all the moments for each connection</td>
</tr>
<tr>
<td>Only Passive Connections</td>
<td>Indicators only at the connections of the receiver (18 GR DOFs)</td>
</tr>
<tr>
<td>Active and passive</td>
<td>Indicators in the source close to the connections and indicators in the receiver (theoretically wrong)</td>
</tr>
<tr>
<td>Only active</td>
<td>Indicators in the source only, close to the connections (theoretically wrong)</td>
</tr>
</tbody>
</table>

Table 1: Sets of DOFs used for the inversion.

The exclusion of Rz is done because of its lower contribution compared with other DOFs. Exclusion of moments is done as well to check negligibility of certain DOFs and the degree of translational DOFs sufficiency when characterizing a lowly damped and rigidly coupled structure.

The inclusion of source indicators pursues the improvement of the condition number and the overdetermination of the system. To reduce the error, the accelerometers of the source are in a region close to the connection that is assumed to behave rigidly together with the receiver side of the connection. Even though strictly speaking this is theoretically wrong, the error made by its inclusion may be lower compared to the potential numerical errors of an ill-conditioned system.
3.2 In-situ validation

The first validation consists of the in-situ validation. The blocked forces are calculated and then used to predict the response in the same structure, in targets that were not used for the inversion, as shown in eq. (24).

\[ F_{bl}^{SR1} = y_{rc}^{SR1} \cdot a_r^{SR1} \rightarrow \sigma_r^{T} = y_{rc}^{SR1} F_{bl}^{SR1} \]  

(24)

Where \( r_T \) refers to the target DOFs. Fig 10 shows a very good match between the measured and predicted target spectra over the entire frequency range. This validation can give an idea of the quality of the blocked forces calculation for the structure but is not really helpful with respect to assess the performance when transferred to a different one. Being the indicators and targets in the same structure, the dominant mechanisms of vibration transmission are very similar, and the matching of the predicted acceleration shows too optimistic results. Moreover, it doesn’t provide any validation for the FBS since the inversion is made using measured coupled FRFs.

![Figure 10: In-situ validation in target T3.](image)

However, already in this validation some differences between the different sets of DOFs explained before can be observed. The results of the Frequency Response Assurance Criterion (FRAC), applied in the whole frequency range to the measured and predicted spectra for every different set of DOFs, are shown in table 2. For instance, the best matches are achieved when using indicators located only in the receiver, whether in the connections or not. While the performance worsens when excluding moments and when using active side indicators. The worst results are obtained if all the moments are excluded and if only active side indicators are used.

<table>
<thead>
<tr>
<th>Label</th>
<th>FRAC value</th>
</tr>
</thead>
<tbody>
<tr>
<td>All indicators</td>
<td>0.9801</td>
</tr>
<tr>
<td>Without Rz</td>
<td>0.9770</td>
</tr>
<tr>
<td>Without Rx Ry Rz</td>
<td>0.7942</td>
</tr>
<tr>
<td>Only Passive Connections</td>
<td>0.9784</td>
</tr>
<tr>
<td>Active and passive</td>
<td>0.9753</td>
</tr>
<tr>
<td>Only active</td>
<td>0.8311</td>
</tr>
</tbody>
</table>

Table 2: FRAC for each set of indicators in the in-situ validation.
3.3 Transferability validation

The second step in the validation is the transferability validation. The blocked forces are calculated in the test bench structure, and then used to predict responses in the vehicle structure, using again measured coupled FRFs, as shown in eq. (25).

\[
F_{bi}^{SR_1} = Y_{rc}^{SR_1}a_{i}^{SR_1} \rightarrow a_{i}^{SR_2} = Y_{rc}^{SR_2}F_{bi}^{SR_1}
\]

Where \( R_1 \) and \( R_2 \) refer to the two different receiver structures. From the figure 11 it can be noticed that the accuracy of the prediction is very good up to 500 Hz, and it lowers in general from 500 Hz toward the high frequencies.

![Figure 11: Transferability validation in target T3.](image)

Again, similarly to the in the in-situ validation, the best results are achieved for the cases using indicators available on the receiver.

As well as for the in-situ validation, excluding moments (figure 12) or including source indicators performs generally worse and at least with no improvement compared to the usage of indicators only in the receiver, observation supported by the FRAC, when evaluated in regions without numerical problems, as shown in table 3.

<table>
<thead>
<tr>
<th>Label</th>
<th>FRAC value</th>
</tr>
</thead>
<tbody>
<tr>
<td>All indicators</td>
<td>0.8475</td>
</tr>
<tr>
<td>Without Rx Ry Rz</td>
<td>0.0275</td>
</tr>
<tr>
<td>Only Passive Connections</td>
<td>0.8626</td>
</tr>
<tr>
<td>Only active</td>
<td>0.7370</td>
</tr>
</tbody>
</table>

Table 3: FRAC for each set of indicators in the transferability validation.
3.4 Transferability with FBS validation

Finally, the transferability of blocked forces using FBS is evaluated as well. The blocked forces are calculated in the test bench structure, and then used to predict responses in the vehicle structure, using coupled FRFs calculated with FBS, as shown in eq. (26).

\[
F_{bl}^{SR_1} = Y_{rc}^{SR_1} a_r^{SR_1} \rightarrow a_r^{SR_2} = Y_{rc}^{R_2} [Y_{cc}^{S} + Y_{cc}^{R_2}]^{-1} Y_{cc}^{S} F_{bl}^{SR_1}
\]  

(26)

In this way, this validation takes into account the effects of the transferability and the FBS coupling. Geometrical reduction is applied to the connections of source and receiver in order to make the coupling, so the rigidity hypothesis will play a determining role in the validation at higher frequencies.

In order to assess the quality of the geometrical reduction the concept of sensor consistency [19] is introduced. This sensor consistency evaluates the similarity between the theoretical rigid behavior and the real measured behavior of the geometrical points where the accelerometers are, for a given force DOF, using a MAC between both acceleration vectors:

\[
\rho^2 = MAC(R_{a}, a) = MAC(\bar{a}, \alpha) = \frac{(\bar{a}^H a)}{(\bar{a}^H \bar{a}) (a^H a)}
\]  

(27)

Where \( \rho^2 \) is the consistency value, ranging from 0 to 1. According to that, a decrease of accuracy would be expected as the frequency increases, which would be added to the effects of the transferability. The product of the consistency for every force DOF is plot in figure 13 for each connection.
A comparison between the measured target acceleration and the predicted values using transferability with FBS is shown in figure 14.

As expected, the highest predictive value is again obtained up to 500 Hz. However, the predicted vibration spectrum seems to be slightly more noisy when compared to the prediction obtained by transferability of blocked forces using measured coupled FRFs, as shown in figure 11.

In this case the differences between the different sets of indicators might be less noticeable due to the accumulated errors in load characterization, geometrical reduction and FBS calculation, and for this reason are not presented.

4 Conclusions

This work has investigated the joint application of techniques that allow the prediction of the behavior of a source in a given receiver, going from source characterization to the calculation of the response. This methodology has already shown some value in similar rigidly connected and low damped structures, in either academic setups [20] or industrial applications [21][22][23]. In this paper the prediction process steps, including the use of FBS equations, have been validated in an academic setup.

For this setup, the vibration prediction using transferability of blocked forces performs slightly better before the FBS application. However, it is found that in the whole frequency range the additional error of the FBS application is considerably lower than the error already made by the calculation of blocked forces and their transferability using measured coupled FRFs. This leads to the conclusion that the main challenge of the
method could be the correct blocked forces estimation (in a stable and complete system) for its transferability, and not mostly the FBS itself. Furthermore, using FBS for blocked forces transferability has the following advantages: i) it allows to couple a source with a receiver not manufactured yet, by means of simulated data of the receiver; ii) it allows to calculate the coupled FRFs of the assembly and the exerted contact forces between source and receiver without being physically coupled.

For this case, theory and experience support consistently the idea of using as many indicators as possible being in the receiver only. In addition, due to its low excitation and contribution, Rz could be neglected from the connection DOFs without detriment to the result, but without a clear improvement either. However, neglecting moments should only happen in the case that the setup and operational condition strongly and reasonably advice to, and not in a general way. As shown in this experimental case of rigid connections, neglecting moments clearly worsen the prediction results.

The accuracy of the prediction is in general good up to 500 Hz, taking into account that is a low damped structure, with a very light and flexible source compared to the receiver, and a rigid connection. Results and conclusions could be carefully extrapolated to similar structures (e.g. steering system) but might vary for others with different conditions of coupling, mass and stiffness (e.g. combustion engine). A future validation of interest would be trying to expand the applicability to assembled structures connected with rubber mounts, which for now remains out of the reach of this work.

Acknowledgements

The authors gratefully acknowledge the European Commission for its support of the Marie Sklodowska Curie program through the ETN PBNv2 project (GA 721615).

References


