Model-based acoustic characterisation of muffler components and extrapolation to inhomogeneous thermal conditions

M. H. Alkmim[1], **J. Cuenca**[1], **L. de Ryck**[1], **P. Göransson**[2]
[1] Siemens Industry Software, Interleuvenlaan 68, B-3001 Leuven, Belgium
e-mail: **mansour.alkmim@siemens.com**

[2] Marcus Wallenberg Laboratory for Sound and Vibration Research, KTH Royal Institute of Technology, Teknikringen 8, SE-10044, Stockholm, Sweden

Abstract

A methodology for evaluating the acoustic behaviour of two-port inhomogeneous media in experimentally unavailable thermal conditions is proposed. The method consists of an inverse estimation of the geometrical and material properties of the object at room temperature followed by a forced thermal input. The properties of interest for the inverse estimation are the spatially-varying cross-section and/or bulk properties. The underlying model relies on a transfer matrix approach, allowing for a representation of spatially inhomogeneous objects as piece-wise equivalent homogeneous fluids, while ensuring continuity conditions between successive elements. A model of non-stationary thermal conduction is used as a first approximation, where an integral formulation accounts for the cumulative effect of multiple homogeneous elements. In order to evaluate the validity of the extrapolation, a validation against a fully numerical simulation is presented in two cases, namely a simple expansion chamber and a complex muffler.

1 Introduction

Mufflers or silencers are commonly used as noise reduction devices for gases transmitted through exhaust pipes. Understanding their properties and behaviour is essential in order to achieve better designs in terms of noise perception and weight reduction. The noise radiated from a muffler arises from three main mechanisms, namely the pulsated pressure waves from the periodic combustion process, the turbulent flow of exhaust gases and structural vibrations, partially caused by the other two sources [1].

A standard procedure to characterize mufflers as acoustic filters is to measure their transmission response from a known acoustic source. The impedance tube setup [2] is particularly attractive as it considers a plane wave approximation, reducing the problem to one spatial dimension. Such tests are well-controlled and repeatable in a limited frequency range; however, these do not represent the operational use of the tested system. Indeed, high-temperature flow of gases denser than air is common in car mufflers, for instance. As a consequence, a major limitation in current laboratory testing is that room temperature sound transmission loss measurements are not representative of most real systems.

Cases where high temperature gradients are encountered such as in automotive mufflers, propulsion and power generating systems benefit from the consideration of thermal effects in the design stage; despite the complexity and cost of such experimental setup. Since the works of Kapur et al. [3], Munjal and Prasad [4] and Peat [5], the study of analytical solutions for plane waves propagation with temperature gradient in straight ducts have been investigated. In most cases, the solutions are restricted to small temperature gradients, no mean flow, small Mach number, among others. To close this gap, Sujith et al. [6, 7] proposed an analytical solution for the sound propagation in ducts under more general conditions by applying suitable transformations to the wave equation. Although the solution therein is valid only for small mean Mach number, the analytical expressions are developed by only considering linear and exponential temperature profiles. Subsequently, by using the analytical solution obtained previously, Sujith [8] derived the transfer matrix method with the axial mean temperature gradient effect included. Later, a correction of inconsistencies therein was proposed by Howard [9] in which a validation against a numerical solution was conducted. Although some progress was made for analytical solutions, its applications are still restricted to regular geometries and boundary conditions. Moreover, real mufflers can feature complex geometries where a simple linear temperature gradient is not enough to predict their behaviour.

As a first approximation, many mufflers can be conveniently described as series of duct components with different cross sections. By assuming plane waves propagating in a rigid-walled tube filled with an inviscid fluid, the so-called transfer matrix method (TMM) [1] conveniently relates the state variables – pressure and flow – from the filter inlet to its outlet. Moreover, the TMM parameters are not affected by connections to external elements downstream or upstream as long as these can be assumed linear [10]. This is advantageous to design mufflers as multiple expansion chambers where the elemental four-pole matrices can be multiplied sequentially to obtain an overall four-pole matrix for the whole system.

This paper addresses the difficulty of performing STL measurements in operational conditions by means of a model-based procedure relying on room temperature STL measurements. The proposed approach consists of an inverse estimation of the geometrical and material model parameters of a two-port inhomogeneous medium, followed by a simulation of its acoustic behaviour in experimentally unavailable thermal conditions. The model relies on the TMM, allowing for a piece-wise representation of spatial inhomogeneity and ensuring continuity conditions between successive elements. The cost function for the inverse estimation procedure is defined as a 2-norm distance between a measured sound transmission loss and its semi-analytical TMM counterpart. A complicating aspect is the fact that such an inverse estimation procedure is subjected to multiple local minima, in particular due to resonances. A recently proposed methodology [11] is used to overcome such local minima. A paper at this conference details its use in the case of muffler components [12].

The properties of interest for the inverse estimation are the spatially-varying cross-section and/or bulk properties of the medium (flow resistivity, porosity, tortuosity, . . .), allowing to represent a variety of one-dimensional objects which can include perforated elements and rigid porous elements which can be modelled as equivalent-fluids. In this paper, however, efforts are concentrated in investigating the temperature extrapolation, hence, only examples considering air as a homogeneous medium are considered, the purpose of the inverse estimation being to identify the geometry of the muffler.

The extrapolation of the acoustical model from room temperature conditions to a forced thermal regime is performed under the assumption that the effects of temperature in the acoustic behaviour arise as a variation in the speed of sound. The temperature profiles can take into account different thermal conditions such as conduction, or a superposition of conduction and convection effects. Two application examples are provided, the goal being to evaluate the validity of the semi-analytical model with temperature extrapolation against a fully numerical simulation in cases including expansion chambers and mufflers.

2 Methodology

2.1 Transfer matrix method

The classical technique to relate the state variables from the inlet and outlet under plane wave assumption is the transfer matrix method (TMM). The TMM or four-pole representation is a convenient way of evaluating acoustical performance because only the four-pole parameters are needed in the analysis of a mufflers as it can readily provide a value for the normal Sound Transmission Loss (STL).

Adopting the notation showing in Figure 1, the transfer matrix formalism using a pressure-flow formulation is given by [1]

$$\begin{bmatrix} p_{\text{in}} \\ S_{\text{in}} v_{\text{in}} \end{bmatrix} = \mathbf{T} \begin{bmatrix} p_{\text{out}} \\ S_{\text{out}} v_{\text{out}} \end{bmatrix}, \tag{1}$$

where p_{in} and p_{out} are the acoustic pressure at the inlet and outlet respectively, v_{in} and v_{out} are the acoustic particle velocity at the inlet and outlet respectively and S_{in} and S_{out} are the cross-sectional area of the inlet (upstream) and outlet (downstream) pipe respectively.

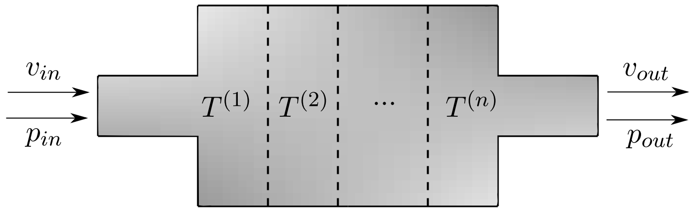


Figure 1: TMM of a multi-layered two-port component using pressure-flow formulation.

The transfer matrix of a multi-layered two-port component can be expressed as

$$\mathbf{T}(\omega) = \prod_{n=1}^{N} \mathbf{T}_n(\omega, \mathbf{x}_n), \tag{2}$$

where the properties of layer n are

$$\mathbf{x}_n = \{L_n, S_n, k_n, Z_n\}, \tag{3}$$

where L_n and S_n are the layer length and cross sectional area respectively. The homogeneous media properties are defined by its wave number k_n and its impedance Z_n. For the case of a homogeneous medium filled with air, both parameters are readily available and can be obtained by the expressions $Z_{air} = \rho_{air} c$ and $k_{air}(\omega) = \omega/c$ where c is the speed of sound and ω the angular frequency. Alternatively, an equivalent fluid model can be used to describe other mediums such as rigid porous and perforated plates. The Johnson-Champoux-Allard (JCA) [13, 14] is a well established phenomenological model that could be used in this case [12].

The transfer matrix of layer n is given by

$$\mathbf{T}_n(\omega, \mathbf{x}_n) = \begin{bmatrix} \cos(k_n L_n) & \dfrac{-Z_n}{iS_n} \sin(k_n L_n) \\ \dfrac{iS_n}{Z_n} \sin(k_n L_n) & \cos(k_n L_n) \end{bmatrix}. \tag{4}$$

The sound transmission loss of the multi-layer component can be obtained as [1]

$$\text{STL} = 20 \log_{10} \left(\frac{1}{2} \left| T_{11} + T_{12} \frac{S_{\text{out}}}{Z_{\text{out}}} + T_{21} \frac{Z_{\text{in}}}{S_{\text{in}}} + T_{22} \frac{Z_{\text{in}}}{Z_{\text{out}}} \frac{S_{\text{out}}}{S_{\text{in}}} \right| \right). \tag{5}$$

2.2 Inverse estimation methodology

The parameter search is formulated as a minimisation problem [15]

$$\min f_{\text{obj}}(\mathbf{x}) = \sum_m |\text{STL}(\mathbf{x}, \omega_m) - \text{STL}_0(\mathbf{x}_0, \omega_m)|^2, \tag{6}$$

where $\mathbf{x_0}$ are the true parameters (accessible by direct measurement only) and $\mathbf{x}$ are the model parameters, including design variables of all layers. $\text{STL}_0(\mathbf{x}_0, \omega_m)$ denotes the measured sound transmission loss (STL) at frequency ω_m and $\text{STL}(\mathbf{x}, \omega_m)$ denotes the STL predicted by the model at frequency ω_m.

The systems under consideration are finite and involve geometrical and material discontinuities, which induce a non-monotonous sound transmission loss with respect to frequency, for instance due to resonances. This results in the problem (6) exhibiting local minima, attributed to the similarity of the response or STL for different combinations of the design variables. The simplest example of this is the similarity of the STL of an expansion chamber whose length is allowed to take values having a common factor. The inverse estimation problem is then solved for $\mathbf{x}$ using a recently developed stepwise optimisation framework [11]. A companion paper at this conference [12] includes an illustration of the method for the inverse estimation of material and geometrical properties using a sound transmission loss measurement setup.

2.3 Heat conduction model and extrapolation

A simplified model is here used for simulating the thermal behaviour of the system when exposed to a constant temperature at the inlet. The model consists of thermal conduction [16] in a multi-layered system whose components consist of equivalent fluid media. The purpose of such a simplified model is to carry out the extrapolation procedure proposed in this paper by means of a analytical description. Although also tractable by analytical means in the case of simple geometries, heat convection and radiation are not considered.

As a hypothesis, the thermal diffusivity of the multi-layer system $\delta(x)$ is assumed to vary slowly in x. The heat equation can thus be written as

$$\frac{\partial \tau}{\partial t}(x, t) = \delta(x) \frac{\partial^2 \tau}{\partial x^2}(x, t), \tag{7}$$

where τ is the temperature, x is the distance along the system and t is time. An imposed temperature is considered at the inlet, as

$$\tau(x = 0, t) = \tau_{\text{in}}. \tag{8}$$

Furthermore, room temperature is assumed at rest and at infinity, which may be expressed as

$$\tau(x, t = 0) = \tau(\infty, t) = \tau_{\text{atm}}. \tag{9}$$

The solution is obtained via the Laplace domain, leading to the temperature profile of a continuously layered medium in the form

$$\tau(x, t) = \tau_{\text{atm}} + (\tau_{\text{in}} - \tau_{\text{atm}}) \operatorname{erfc}\left(\frac{1}{2\sqrt{t}} \int_0^x \frac{d\xi}{\sqrt{\delta(\xi)}} \right). \tag{10}$$

This expression thus defines the temperature profile as a function of the input temperature, ambient temperature and the thermal diffusivity profile. It is worth noting that for a system consisting of a finite number of homogeneous layers, the thermal diffusivity is piecewise constant and the integral reduces to a step function. The obtained temperature profile is then introduced in the speed of sound in air as

$$c(x) = \sqrt{\gamma R \tau(x)}, \tag{11}$$

where $\gamma = 1.4$ is the ratio of specific heats and $R = 287\,\text{J}\cdot\text{K}^{-1}\text{mol}^{-1}$ is the gas constant.

3 Numerical validation

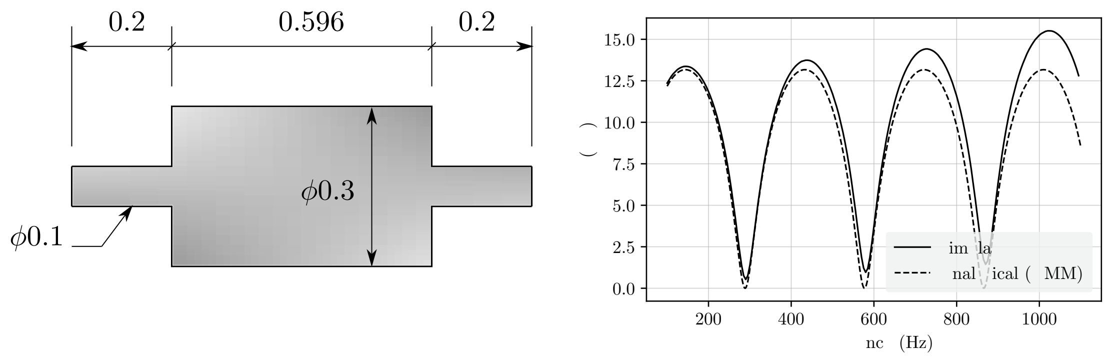
The aim of this section is to validate the thermal extrapolation technique against a fully numerical simulation in order to assess the quality of the inverse estimated model. Two cases are considered: (3.1) single expansion chamber and (3.2) complex muffler formed by multiple expansion chambers. The numerical simulation is performed using Simcenter 12.0 for meshing and thermal-flow simulation. With the obtained temperature field the sound speed can be calculated for each element. The acoustic finite element simulation is performed using LMS Virtual.lab 13.8. The simulation was performed considering the frequency range from 100 Hz to 1100 Hz with a 5 Hz step. The environmental conditions are conventional, with atmospheric pressure at 101351 MPa and ambient temperature at 293 K. The boundary conditions for the thermal-flow simulation considers a slow flow condition with inlet flow $v = 0.001$ m/s and external condition given by an imposed temperature of 573 K. The outlet flow has the same flow velocity. A mixing length turbulence model for thermal-flow solver is employed. The duct wall consider is Aluminium A356 with thickness of 2 mm and tetrahedral shape (TET4) with element size of 51.5 mm for the single expansion chamber and 25 mm for the complex muffler. For the acoustic meshing, the tetrahedral shape (TET4) with same element size of their respective thermal simulation are used for mapping convenience. Isotropic air is used as the material which contains the properties from Table 1.

Table 1: Air properties at ambient temperature

Mass Density	$\rho = 1.2041\ \mathrm{kg \cdot m^{-3}}$
Speed of Sound	$c = 343.21\ \mathrm{m \cdot s^{-1}}$
Thermal Conductivity	$\kappa = 0.023\ \mathrm{W \cdot mm^{-1} \cdot K^{-1}}$
Isobaric specific heat capacity	$c_p = 1.005\ \mathrm{kJ \cdot kg^{-1} \cdot K^{-1}}$

3.1 Single expansion chamber

The expansion chamber has the same dimensions as described in [17] and reproduced in Figure 2 (a). Figure 2(b) shows the simulated STL of the single expansion chamber and a STL calculate analytically using TMM at ambient temperature for reference.



(a) (b)

Figure 2: (a) Dimensions of expansion chamber in m and (b) STL of expansion chamber comparing a simulated result with analytical using TMM at ambient temperature.

It can be noted from Figure 2 that the simulated STL does not match the analytical STL. The discrepancy between the STL increases as the frequency increase. This occurs because the analytical model utilizes a one-dimensional propagation approximation (plane wave assumption) which does not account for attenuation and it also does not capture higher order modes. This limitation of such a one-dimensional model is well known [1]. As a matter of fact, the three-dimensional effects play a role well below the cut-on frequency of the first higher order mode. Taking into account the limitations of the TMM, the inverse estimation approach only considers the values for the STL for frequencies up to 300 Hz. The obtained retrieved geometry of the expansion chamber can be seen in Table 2 and in Figure 3.

Table 2: Dimension of original and retrieved expansion chamber

	Diameter (m)	Length (m)
Original	0.300	0.596
Retrieved	0.304	0.591

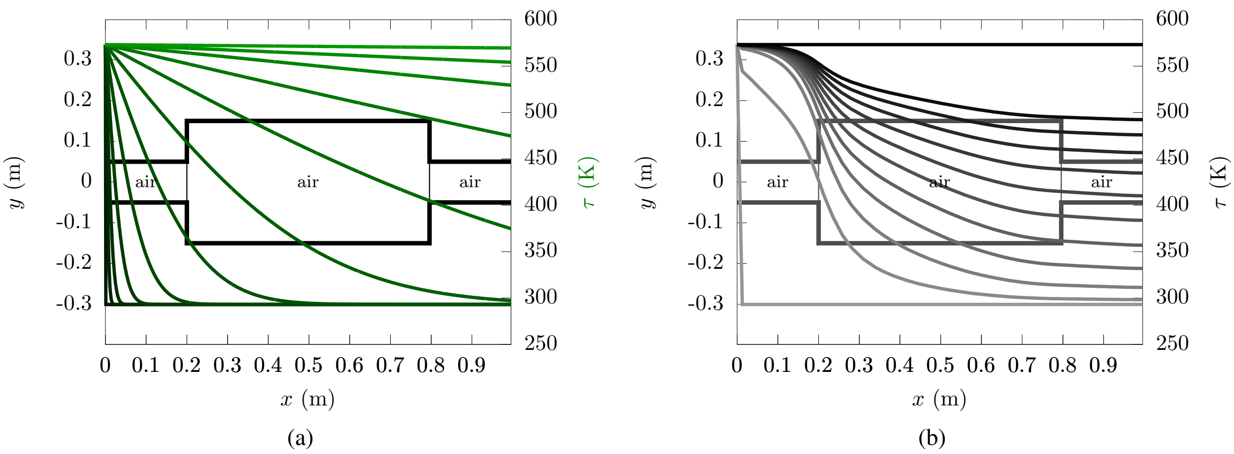


Figure 3: Retrieved geometry and the two transient temperature profiles considered: (a) from heat conduction model and (b) from duct centerline in the performed thermal-flow simulation.

The retrieved values show good agreement with the original dimension and it gives confidence to tackle the temperature extrapolation problem. The two temperature extrapolation scenarios are shown in Figure 3. The profiles from Figure 3 (a) are obtained from the heat conduction model from Section 2.3 and the profiles in Figure 3 (b) are extracted from the expansion chamber centerline in the performed thermal-flow simulation.

Although the initial conditions for the two temperature extrapolation scenarios are the same, a considerable difference in temperature profiles can be seen in Figure 3 where it is possible to observe the transient evolution of the temperature profiles along the system. In Figure 3(a) the first profiles show a very abrupt temperature change and gradually, as time increases, the outlet temperature increases and the profiles become almost linear. This behaviour is due to the simplified thermal model, which does not include the effect of different cross section areas. On the other hand, the simulated scenario from Figure 3(b) shows a less abrupt and a more gradual outlet temperature change between each profile, as it takes into account cross-section variations.

In this simple case with symmetrical geometry, the temperature profile can be easily obtained, which is not the case for complex geometries. For this reason, both the simulated temperature profiles and the heat conduction model temperature profiles are considered for the acoustic performance calculation that follows.

Figure 4 shows the comparison between semi-analytical and FEM-simulated STL considering the heat conduction model and the FEM temperature profiles. The selected profiles are as follows: (a) constant temperature $\tau = 293$ K, (b) - (e) intermediate profiles with boundary condition that can be referred at Table 3 and (f) at constant $\tau = 573$ K. A comparison between the profiles shows a difference in attenuation levels, mostly in the intermediate profiles. Besides, a shift in the STL frequency domain due to the temperature change can be clearly observed for the intermediate profiles in Figure 4 and less noticeable in Figure4. This frequency shift has a relevant impact in the region of the first dip (or lobe). This phenomenon is critical as it can change the location of optimal sound transmission loss content, alter the muffler's performance and consequently, audio perception. The quantification of this frequency shift is shown in Figure 5 where, the difference between analytical (TMM) with heat conduction and FEM-simulated profiles is compared with the fully simulated STL. For the difference between analytical (TMM) with heat conduction profile and fully simulated STL (green curve), It can be seen a considerably increase in the shift as it approaches intermediate profiles having its maximum value for the profile 7. The large frequency shift in this case is expected because the comparison with the simulated profiles does not match perfectly. However, for small temperature gradients (profiles 9 to 12) the error is considerably small. For the difference between analytical (TMM) with simulated profile and fully simulated STL, the frequency shift is within the resolution of the frequency domain which is 5 Hz. This shows that the analytical model can yield good results compared to the FEM simulation, and the choice of thermal model has a significant impact in the acoustic behaviour. In cases where the temperature profiles are unavailable, the heat conduction model could be used for small temperature gradient.

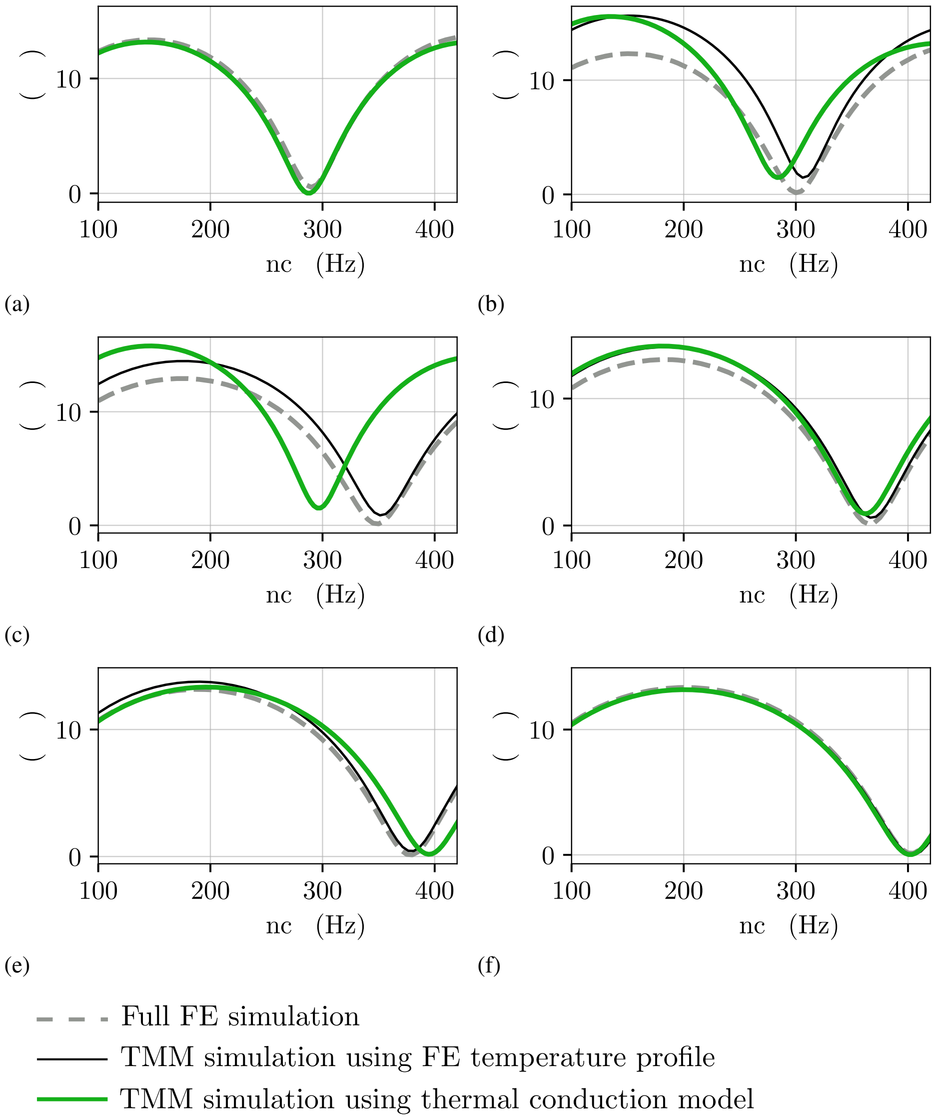


Figure 4: Comparison of single expansion chamber STL between the two methods, analytical (extrapolation) and FE simulation, for the heat conduction model and simulated temperature profiles where (a) is at constant $\tau = 273$ K, (b) - (e) are intermediate profiles (refer to Table 3) and (f) is at constant $\tau = 573$ K.

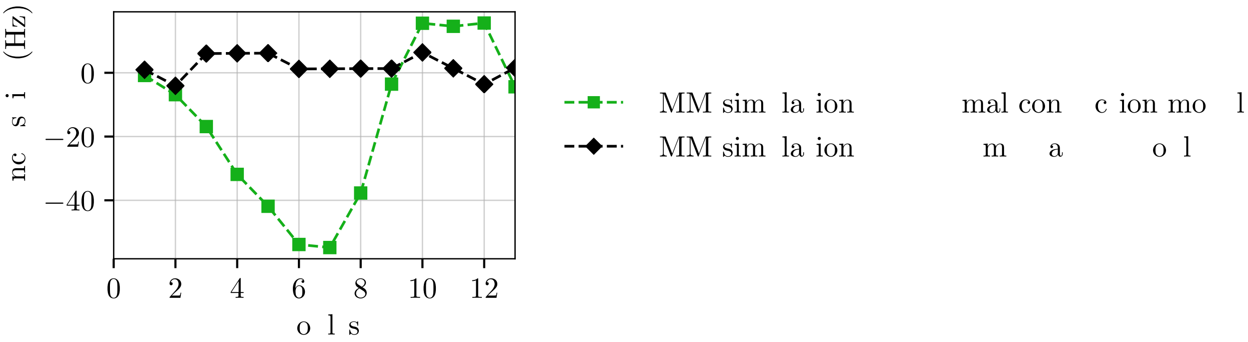


Figure 5: Frequency shift at the STL dip with respect to the full FE simulation considering the TMM simulation with thermal conduction model and with FE temperature profile.

Table 3: Boundary conditions for each profile ($\tau(0)$ - $\tau(L)$, where L is the total length of the system).

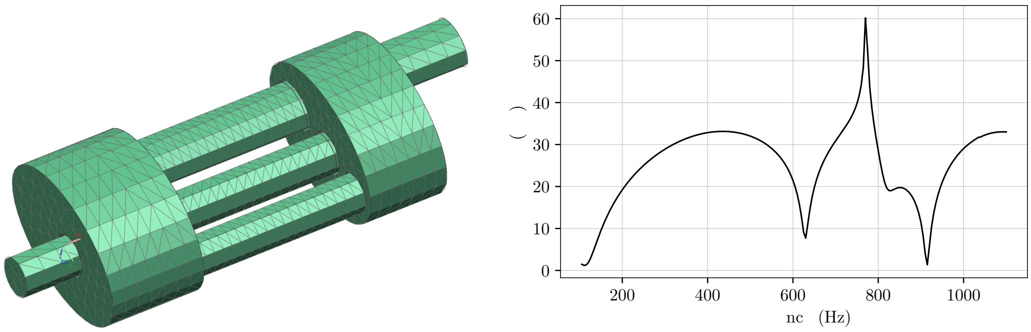
Profiles	**Fig 4.**	$\tau_{\text{conduction}}$ [K]	τ_{FEM} [K]
profile 1	(a)	293-293	293-293
profile 2	-	573-293	573-293
profile 3	(b)	573-293	573-300
profile 4	-	573-293	573-320
profile 5	-	573-293	573-335
profile 6	-	573-293	573-355
profile 7	(c)	573-293	573-380
profile 8	-	573-300	573-410
profile 9	(d)	573-375	573-440
profile 10	-	573-475	573-460
profile 11	(e)	573-530	573-475
profile 12	-	573-553	573-495
profile 13	(f)	573-573	573-573

3.2 Complex muffler

In this section, a similar analysis is made for a complex muffler geometry. The main objective here is to reconstruct a one-dimensional profile that behaves in an equivalent manner as the muffler with a complex geometry. Due to the non-uniqueness of the one-dimensional path connecting the inlet to the outlet, a simple heat conduction model temperature profile is considered as a first approximation. The complex muffler consists of three ducts with two intermediate expansion chambers, as shown in Figure 6 (a). Figure 6 (b) shows the simulated STL at ambient temperature. As it can be seen, a peak in amplitude occurs at around 760 Hz which represents a transversal mode. The region of main interest, however, is in the first lobe which occurs around 610 Hz.

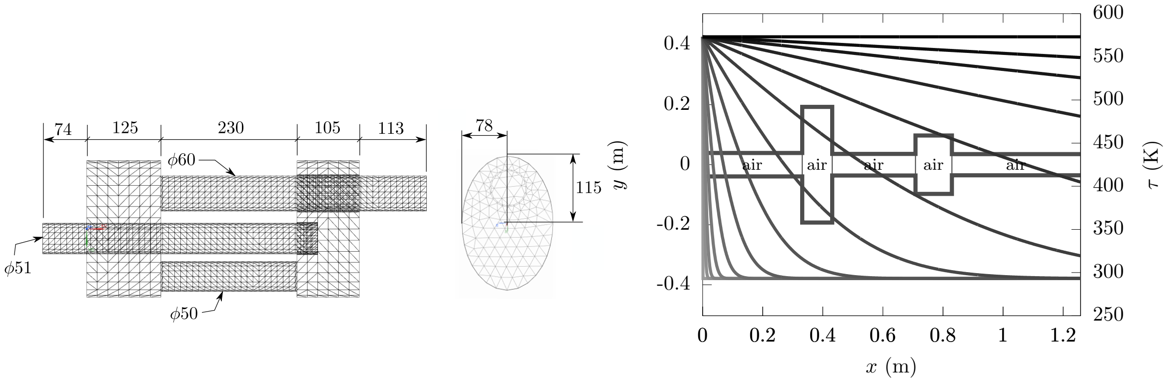
Based on this observation and to avoid second modes in the optimization scheme, only frequencies up until 700 Hz are considered in the inverse estimation. The inverse estimation retrieve an equivalent geometry as shown in Figure 7 with exact dimensions for each section described in Table 4. Although it is quite challenging to compare the original and retrieved geometry, the retrieved geometry does resemble the original muffler since both the complex muffler and the retrieved ones are basically a combination of multiple connections and expansions.

Figure 8 shows the comparison between analytical STL considering the same heat conduction model temperature profiles as previously and a fully simulated STL . The quantification of frequency shift of the dip



(a) (b)

Figure 6: (a) Simcenter 3D model of complex muffler and (b) simulated STL at ambient temperature.



(a) (b)

Figure 7: (a) 3D geometry of complex muffler (dimensions in mm) and (b) retrieved equivalent 1D geometry and imposed temperature profiles.

Table 4: Retrieved complex muffler dimensions for each section

Sections	1	2	3	4	5
Length (m)	0.2343	0.1000	0.1077	0.2773	0.2140
Radius (m)	0.0400	0.1146	0.0230	0.1056	0.0190

can be seen in Figure 9. As expected, there is barely a shift for the constant temperature. The shift error increases considerably as it approaches the intermediate profiles, it been the highest at profile 8. For very high gradient and very small gradient there is a relatively good match. However, the intermediate profiles result in a considerably frequency shift and are not very representative of the acoustic behaviour of the muffler. For this case, a simple heat conduction consideration is not enough to model the thermal behavior of a complex muffler and a more representative model is needed.

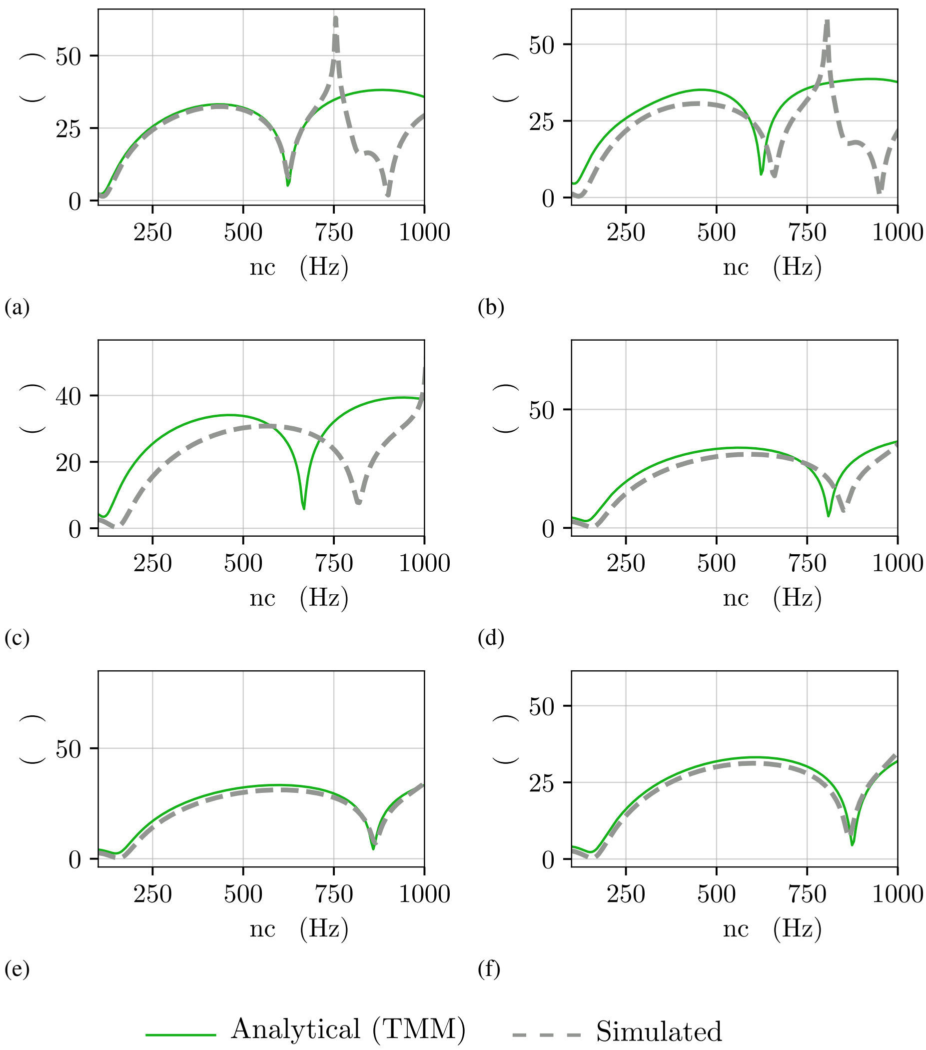


Figure 8: Comparison of the complex muffler STL between the two analytical model with heat conduction model temperature profiles and the fully simulated model where (a) is at constant $\tau = 273$ K, (b) - (e) are intermediate profiles (refer to Table 3) and (f) is at constant $\tau = 573$ K.

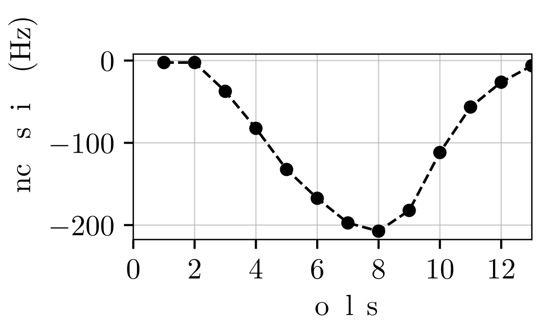


Figure 9: Frequency shift at the STL dip considering the difference between analytical (TMM) with heat conduction compared to the fully FEM simulated model.

4 Conclusion

This paper is concern with the acoustic characterization of mufflers to inhomogeneous thermal conditions. The STL is sensitive to thermal conditions; therefore standard room-temperature measurements are often not representative of operating conditions. Model extrapolation allows to simulate a laboratory object in operating thermal conditions, or to retrieve ISO-comparable tests from thermally-inhomogeneous measurements. Possible applications include experiment-based design of multi-layer media, for instance for performance optimisation in realistic conditions. Experimental and three-dimensional numerical validations are required in order to determine the range of validity of the extrapolation procedure. Further work would require a refined model of thermal processes in porous media.

From a classical or standard STL measurement in impedance tube, two extra steps are added in order to extrapolate the laboratory results to operational conditions unavailable for precise testing. The first step concerns the inverse estimation on the STL itself to retrieve an equivalent geometry for complex components. One advantage of the equivalent geometry is that it allows for a easy and fast way to extrapolate the model to new conditions and a feasible alternative to fully numerical simulations. The method could retrieve the original geometry successfully for the simple case of an expansion chamber. For the more complex muffler studied the equivalent geometry with axial symmetry included some of the relevant characteristics of the original object. The second step involves the temperature extrapolation as such. The latter was compared with finite elements numerical simulations and it showed promising results. Indeed, the main effect sought in the extrapolation is the frequency shifting of the acoustic filter response, like the resonances from the studied expansion chambers. This part is induced by the temperature effect on the sound speed. Nevertheless, the dependency on cross sections seems to play an important role in the thermal model, as seen in the finite element model, which requires further investigation for a more accurate (semi) analytical model. Finally, the attenuation or gain remains to be modeled within this extended but still simple physical model.

Acknowledgments

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